

*Research project DRPK3***Density of rational points on K3 surfaces**

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Research project short description:

The subject of this project is the density of rational points on K3 surfaces over the field of rational numbers. In the classification of surfaces, K3 surfaces lie on the border between geometrically simple surfaces and geometrically complicated surfaces. On the simpler ones, which include the so called Del Pezzo surfaces, the rational points are expected to be abundant. In many cases this is proved; the Segre–Manin Theorem, for instance, states that the rational points on a Del Pezzo surface of degree at least 3 are Zariski dense as soon as there is at least one rational point. On the other hand, on the more complicated surfaces the rational points are expected to all be contained in some finite number of curves, according to Lang’s Conjecture. In the same line, we expect the geometry of K3 surfaces to govern their arithmetic. Bogomolov and Tschinkel [BT] have proven that if an important geometric invariant of a K3 surface X over a number field K , its so called Picard number, is big enough, then there is some finite field extension L of K such that the set of L -points on X is dense. On the other end of the spectrum, this project concerns the density of rational points on K3 surfaces with low Picard number, in particular of Picard number 1, the smallest possible. The literature currently contains only one specific example of a K3 surface over a number field with Picard number 1 on which the rational points are claimed to be dense [K]. One specific subproject would investigate whether the arguments used for that example can be used to find an easier example over the field of rational numbers. There are more approaches that could form the base of other subprojects.

References:

[BT] F.A. Bogomolov and Y. Tschinkel, *Density of rational points on elliptic K3 surfaces*, Asian J. Math. **4** (2000), no. 2, 351–368.

[K] I. Karzhemanov, One construction of a K3 surface with the dense set of rational points, preprint.