

*Research project***Types and lattices in representations of $\mathrm{GL}(3, F)$, F a p -adic field.**

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Research project short description:

Let F be a p -adic field, and G a reductive group over F . The p -adic Langlands program, still in its infancy, seeks to relate p -adic (Banach, or locally analytic, or locally algebraic) representations of $G(F)$ and Galois parameters of some kind. A particular role is played by invariant lattices, which give rise to a reduction mod. p for such representations. Other important ingredients are types: if K is a maximal compact subgroup of $G(F)$, types are irreducible representations of K which are “typical” in the restriction to K of an irreducible representation of $G(F)$.

The goal of the thesis would be to investigate the case of $\mathrm{GL}(3, F)$, and to try and get in that case results similar to known results for $\mathrm{GL}(2, F)$. For example, when $p > 3$, is there a uniqueness result for typical representations of $K = \mathrm{GL}(3, \mathcal{O}_F)$ in the restriction of a given irreducible smooth representation r of $\mathrm{GL}(3, F)$? If p is 2 or 3, and such uniqueness does not hold, can one say at least that the reduction mod. p of the different typical representations in r have isomorphic semisimplification? (See reference [He] for the case of $\mathrm{GL}(2, F)$; the case of reduction modulo a prime ℓ distinct from p should also be investigated). Finally, invariant lattices in r should be investigated, at least in some cases. For instance, if r is a principal series representation of $\mathrm{GL}(3, \mathbf{F}_p)$, can one describe (as in reference [Br] for $\mathrm{GL}(2, \mathbf{F}_p)$) the trace on (some) typical representations of the invariant lattices given by a global model?

Related questions should also be investigated as they present themselves.

References:

[He] G. Henniart, appendix to C. Breuil and A. Mézard, Multiplicités modulaires et représentations de $\mathrm{GL}_2(\mathbf{Z}_p)$ et de $\mathrm{Gal}(\overline{\mathbf{Q}}_p/\mathbf{Q}_p)$ en $\ell = p$, Duke Math. J. 115, 2002, 205-310.

[Pa] V. Paskunas, Unicity of types for supercuspidal representations of GL_N , Proc. London Math. Soc. 91, 2005, 623-654.

[Br] C. Breuil (with an appendix by L. Dembélé), Sur un problème de compatibilité local-global modulo p pour GL_2 , preprint, available at:
<http://www.math.u-psud.fr/~breuil/PUBLICATIONS/compamodp.pdf>