Research project **GCCA**

The Galois closure of commutative algebras

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Research project short description:

For a map $R \to A$ of commutative rings, with A locally free of finite constant rank n over R, Bhargava has defined a "Galois closure" $G(A/R)$, which is a certain commutative R-algebra equipped with a sequence of n R-algebra homomorphisms $A \to G(A/R)$. The project is concerned with the study of this notion, which is of a fairly mysterious nature.

Several explicit questions are raised in Bhargava's preprint. Here are a few others.

(a) It is probably easy to show that the n natural maps $A \to G(A/R)$ are non-zero. Are they injective? Are they pairwise different?

(b) Let A' be the subring of $G(A/R)$ generated by the image of the first two maps $A \to G(A/R)$. Give conditions guaranteeing that A' is locally free as an A-module and for $G(A'/A)$ to be isomorphic to $G(A/R)$ as an A-algebra. Can one describe the kernel of $A \otimes_R A \to A'$?

(c) If A is free as an R-module, and finite étale over R, is $G(A/R)$ free as an R-module? (It is know to be locally free.)

(d) If A_0 and A_1 are two A's, can one describe $G(A_0 \times A_1/R)$ and $G(A_0 \otimes_R A_1/R)$ in terms of $G(A_0/R)$ and $G(A_1/R)$?

(e) If A is a complete intersection over R, is $G(A/R)$ locally free of rank $n!$ as an R-module?

References:

M. Bhargava, M. Satriano, *On the notion of "Galois closure" for extensions of rings*, preprint, October 4, 2009.