

*Research project TPECBD***Torsion points on elliptic curves over number fields of given degree**

Supervisor 1 Bas Edixhoven  
E-mail [edix@math.leidenuniv.nl](mailto:edix@math.leidenuniv.nl)  
Institution Universiteit Leiden

Supervisor 2 Pierre Parent  
E-mail [Pierre.Parent@math.u-bordeaux1.fr](mailto:Pierre.Parent@math.u-bordeaux1.fr)  
Institution Université de Bordeaux 1

Supervisor 3 Bert van Geemen  
E-mail [lambertus.vangeemen@unimi.it](mailto:lambertus.vangeemen@unimi.it)  
Institution Università degli studi de Milano

**Research project short description:**

For  $d$  be a positive integer, let  $S(d)$  be the set of prime numbers that occurs as the order of a rational point of an elliptic curve over a number field of degree at most  $d$  over  $\mathbf{Q}$ . It is known by work of Merel that all sets  $S(d)$  are finite. For  $d \leq 4$  the sets  $S(d)$  are known explicitly by work of Mazur ( $d = 1$ ), Kamienny ( $d = 2$ ), Parent ( $d = 3$ ) and Kamienny, Stein, and Stoll ( $d = 4$ ).

The aim of this project is to extend this work to a few more  $d$ .

The main task of the candidate will be to make the methods by which formal immersion criteria are verified more efficient, so that higher values of  $d$  can be treated. This involves computational aspects as well as theoretical ones. The candidate will also carry out the necessary computations, using computers.

The candidate will spend most of the time in Leiden, Bordeaux, and Milano, but will also collaborate with Stoll (Bayreuth, Germany) and Stein (Seattle, USA) and visit them.

**References:**

- Bas Edixhoven. *Rational torsion points on elliptic curves over number fields (after Kamienny and Mazur)*. Séminaire Bourbaki, Vol. 1993/94. Astérisque No. 227 (1995), Exp. No. 782, 4, 209–227.
- Sheldon Kamienny. *Torsion points on elliptic curves and  $q$ -coefficients of modular forms*. Invent. Math. 109 (1992), no. 2, 221–229.
- Sheldon Kamienny, William Stein and Michael Stoll. *Torsion points on elliptic curves over quartic number fields*. In preparation.

- Barry Mazur. *Modular curves and the Eisenstein ideal*. Inst. Hautes Études Sci. Publ. Math. No. 47 (1977), 33–186 (1978).
- Barry Mazur. *Rational isogenies of prime degree (with an appendix by D. Goldfeld)*. Invent. Math. 44 (1978), no. 2, 129–162.
- Loïc Merel. *Bornes pour la torsion des courbes elliptiques sur les corps de nombres*. Invent. Math. 124 (1996), no. 1-3, 437–449.
- Pierre Parent. *Bornes effectives pour la torsion des courbes elliptiques sur les corps de nombres*. J. Reine Angew. Math. 506 (1999), 85–116.
- Pierre Parent. *Torsion des courbes elliptiques sur les corps cubiques*. Ann. Inst. Fourier (Grenoble) 50 (2000), no. 3, 723–749.
- Pierre Parent. *No 17-torsion on elliptic curves over cubic number fields*. J. Théor. Nombres Bordeaux 15 (2003), no. 3, 831–838.