Diophantine aspects of Automatic Sequences

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A project for an Algant PhD

Finite automata are a powerful tool for tackling properties of numbers through their expansion in a given base b. A very complete exposition is to be found in [4].

Some famous conjectures in the field of real Diophantine analysis, which are, up to now, untractable have natural counterparts which are more accessible in a function field setting. To quote one example: we do not know whether the sequence $((3/2)^n)_n$ is uniformly distributed modulo 1, but we have some knowledge of the distribution modulo 1 of the sequence (θ^n), when θ is an element of $\mathbb{F}_q((X))$ [the field of Laurent series over the finite field with q elements $\mathbb{F}_q$] which is algebraic over $\mathbb{F}_q(X)$ (cf. [5], [3], [2]). Although we know that in a weak ("logarithmic") sense there is a limit distribution, it remains to determine whether a limit distribution exists in the strong sense, what is the nature of the limit distribution, for example is it "pure" in its Lebesgue decomposition? can the atomic component be different from the Dirac measure at the origin?...

Initiated by G. Dresden [7], the study of the least significant digit $\ell_b(n!)$ of $n!$ in a general base b is not yet closed. When $b = 10$, the sequence $(\ell_{10}(n!))_n$ is 5-automatic, whereas for some special values of b (for example $b = 12$, studied in [6]) the sequence $(\ell_b(n!))_n$ coincides for most of its values with an automatic sequence, but possibly not on all values: it is a point to be proved which leaves the door opened to show that all integers in $\{1, \ldots, b-1\}$ occur infinitely often in the sequence, contrary to what happens when $b = 10$.

B. Adamczewski and Y. Bugeaud [1] recently solved the Cobham-Loxton-van der Poorten conjecture that the irrational automatic numbers are transcendental. We offer to explore more general constructions of transcendental numbers out of automatic numbers.

References

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